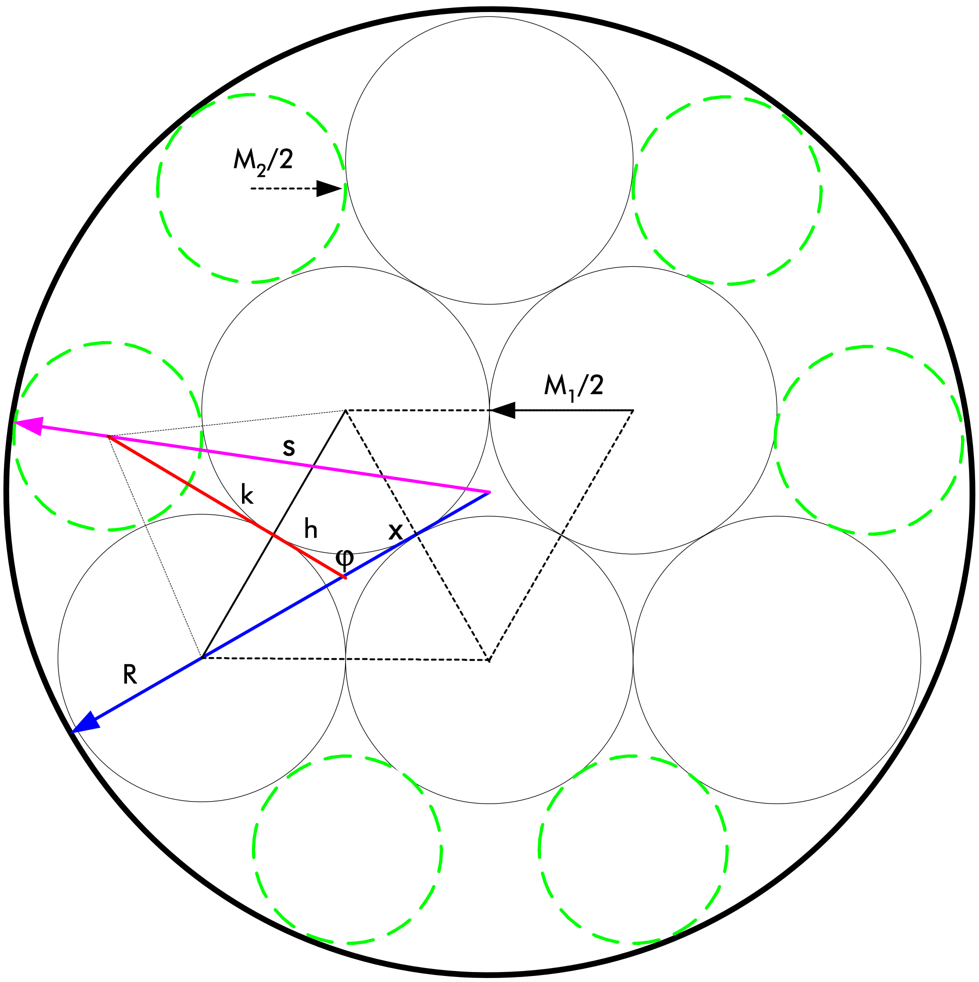
The Geometry of Engine Clusters



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Foreword

Engine clusters have always been an exciting facet of model rocketry, and almost all rocketeers have built a clustered rocket at one point or another in their modeling career. Invariably, the question arises as to the engine combinations that might fit into a particular airframe, and as a matter of expediency rocketeers will often resort to empirical methods to answer this question. While doing so can be convenient for small diameter rockets, the attempt can become rather unwieldy for larger airframes, or for complex cluster arrangements. Given that the cluster possibilities are essentially bounded once an airframe is selected, it should be possible to calculate the engine sizes/cluster combinations that can fit into the airframe. It was from this consideration that this paper was initially prepared.

The original version of this paper examined the geometry of engine clusters arranged in regular polygonal layouts (e.g.: triangle, square, pentagon, etc). The analysis found a set of general expressions that relate motor mount and cluster size to airframe diameter, permitting the modeler to finalize his cluster design well before he must reach for an airframe. Most rocketeers will find the results from the original analysis more than adequate for most of their typical clustering needs.

This updated, second revision looks beyond the primary cluster arrangement and considers the more complex geometry that occurs when mixed engine types are deployed in the cluster. This second revision was inspired by fellow rocketeer Doug Sams, whose excellent empirical work on complex cluster layouts can be found at http://home.flash.net/~samily/stuff/Clusters.pdf.

Because of the amount of new material, the paper has now been divided into two parts. Part I presents the cluster analysis provided in the original paper, together with a few minor updates; Part II presents the new work dealing with the analysis of complex clusters.

The analysis is not intended to be exhaustive; engine cluster combinations are limited only by the size of a particular airframe and the modeler's imagination. Nevertheless, it is hoped that rocketeers will find the results of this examination helpful, and a useful input into their designs for clustered rockets.

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March 2006

Part I:

Regular Polygonal Clusters

1.0 Introduction

One of the interesting and challenging aspects of model rocketry is successfully designing and operating clustered motor rockets. The motivation for building a clustered rocket might be to emulate a scale prototype, or to increase impulse, or perhaps to just challenge one's self to experiment with a new aspect of the hobby.

Regardless of the reason, once the decision is taken to build a cluster, the modeler must immediately decide the cluster arrangement for the airframe of choice. This paper examines the geometry associated with engine clusters, and develops a set of relations that can be used to determine workable engine cluster and airframe combinations, based on commercially available components.

1.1 Definition of Terms

AF_D: The minimum inside diameter of an airframe that will just accommodate the motor cluster.

CV_D: The Central Void, the space formed in the middle between engines arranged in a cluster.

M_D: The outside diameter of the Motor Mount Tube.

'n': The number of sides in an n-sided polygon; also the number of engines in a cluster.

Regular Polygon: A closed, 2-dimensional figure possessing sides of equal length and interior angles of equal value.

1.2 Summary of Results

The analysis shows that the following relations hold true for engines arranged in regular polygonal clusters:

Airframe Diameter:

$$AF_D \geq M_D\left[\frac{1}{\sin\left(\frac{180^\circ}{n}\right)}+1\right]$$

Where $n \geq 2$

Central Void Diameter:

$$CV_D \leq M_D\left[\frac{1}{\sin\left(\frac{180^\circ}{n}\right)}-1\right]$$

Where $n \geq 2$

Number of Engines, "n":

$$n = INT\left\{\frac{180^\circ}{\arcsin\left[\frac{M_D}{AF_D - M_D}\right]}\right\}$$

Where INT = the Integer function;

And $AF_D > M_D$

Summarized below are the key parameters for the first few polygonal cluster arrangements:

Table 1: Polygonal Cluster Summary

# Of Engines	Cluster Arrangement	$AF_D \geq$	$CV_D \leq$
2	Pair	2 x M_D	No Void
3	Triangle	2.155 x M_D	0.155 x M_D
4	Square	2.414 x M_D	0.414 x M_D
5	Pentagon	2.701 x M_D	0.701 x M_D
6	Hexagon	3 x M_D	1 x M_D
7	Heptagon	3.305 x M_D	1.305 x M_D
8	Octagon	3.613 x M_D	1.613 x M_D

2.0 The 2-Engine Cluster

An engine cluster is created the instant two or more engines are combined in a non-staged configuration. In Figure 1, two motor mount tubes are mounted inside the airframe of a rocket, defining the simplest minimum cluster arrangement.

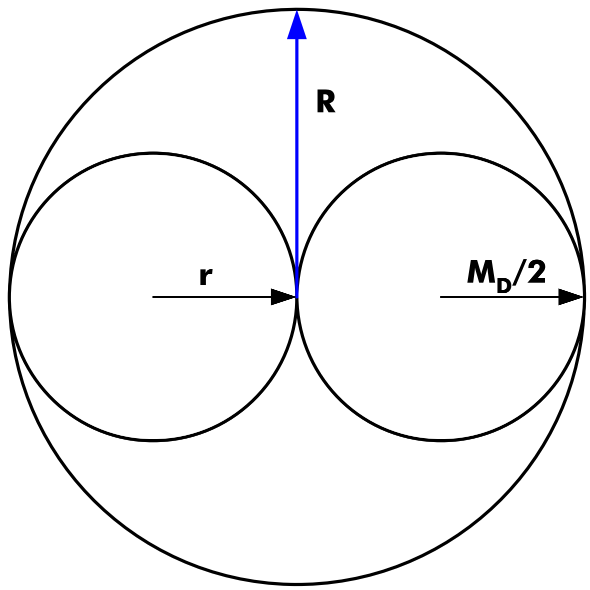

Figure 1: 2-Engine Cluster

Each motor mount is defined by its radius r; in this case the two motor mounts are of equal diameter, which is the typical case in most simple cluster arrangements. The airframe is defined by its radius R, and is represented by the blue line in the diagram. For the ensuing discussion we will find it more convenient to speak in terms of the diameter of these components, so we define the following relationships:

$M_D = 2r$; where M_D is the outside diameter of the Motor Mount Tube.

$AF_D = 2R$; where AF_D is the minimum inside diameter of the airframe the cluster will just fit into.

What we would like to find is a formula that relates the minimum inner diameter of the airframe to the diameter of the motor mount tubes. Knowing this, we can quickly calculate the stock tube combinations that will accommodate the motor combinations we might wish to use in the rocket design.

In this particular cluster example, we can readily see that the minimum inside diameter of the airframe must be at least equal to or greater than $2M_D$. And so we have the following relationship for the 2 Engine Cluster:

$AF_D \geq 2M_D$

3.0 The 3-Engine Cluster

Clustering three engines of equal diameter creates the shape of an equilateral triangle, the vertices of which are located at the center of each engine. Figure 2 illustrates this arrangement.

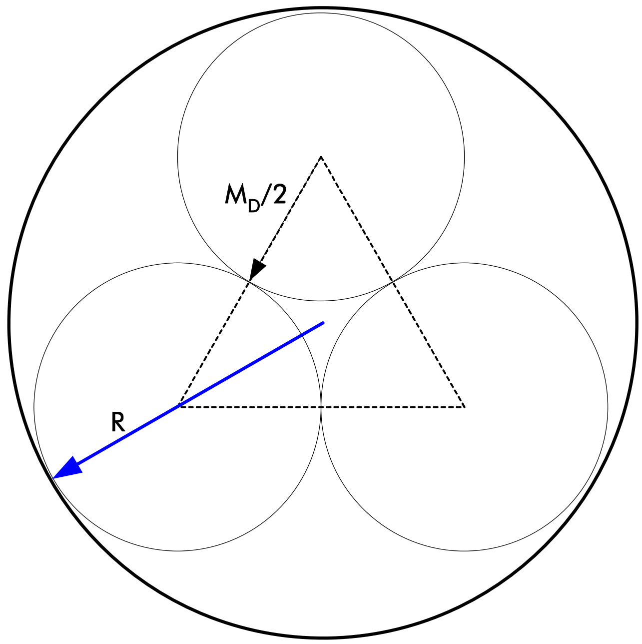

Figure 2: 3-Engine Cluster

We will now focus on the geometry formed by this arrangement. Figure 3 strips away the tubes and simply presents the geometry.

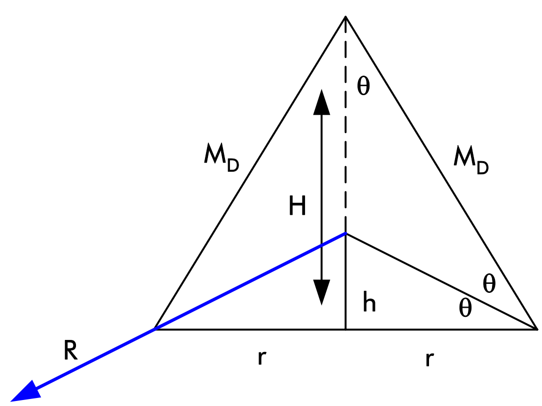

Figure 3: 3-Engine Cluster Geometry

We readily recognize the key property of an equilateral triangle, the sides being of equal length. In this case, the sides are each 2r, where r = the radius of the motor mount tube.

We will now declare the following definitions. Let:

- **2r** = M_D.
- **R** = the minimum inner Airframe Radius, as represented by the blue line in the figure.
- **H**= the height of the equilateral triangle.
- **h** = the perpendicular distance from the midpoint of any side to the center of the triangle.
- **(H-h)** = the distance from the center of the triangle to any vertex.

The minimum inner radius of the airframe **R** needed to accommodate this cluster is equal to the distance from the center of the triangle to a vertex, plus an additional distance of r, the radius of the motor tube. We'll express this length as follows:

$$R = (H - h) + r$$

But $r = \frac{M_D}{2}$

So $R = (H - h) + \frac{M_D}{2}$

And since $AF_D = 2R$, then:

$$AF_D \geq 2(H - h) + M_D$$

Applying the Pythagorean Theorem to the smaller internal triangle, we can see that:

$$(H - h)^2 = h^2 + r^2 = h^2 + \frac{M_D^2}{4}$$

Looking at the larger Right Triangle, we can see that:

$$H^2 + r^2 = H^2 + \frac{M_D^2}{4} = M_D^2$$

$$\therefore H^2 = M_D^2 - \frac{M_D^2}{4} = \frac{3M_D^2}{4}$$

And so $H = \frac{\sqrt{3}M_D}{2}$

Substituting this result into the earlier relation for the smaller internal triangle, we get:

$$\left(\frac{\sqrt{3}M_D}{2} - h\right)^2 = h^2 + \frac{M_D^2}{4}$$

$$\frac{3M_D^2}{4} - \sqrt{3}M_D h + h^2 = h^2 + \frac{M_D^2}{4}$$

$$\frac{M_D^2}{2} = \sqrt{3}M_D h$$

$$\therefore h = \frac{M_D}{2\sqrt{3}}$$

$$\therefore (H - h) = \frac{\sqrt{3}M_D}{2} - \frac{M_D}{2\sqrt{3}}$$

$$= \left[\frac{3-1}{2\sqrt{3}}\right]M_D$$

$$= \frac{M_D}{\sqrt{3}}$$

Since $AF_D \geq 2(H - h) + M_D$

Then $AF_D \geq \frac{2M_D}{\sqrt{3}} + M_D$

$$\geq \left[1 + \frac{2}{\sqrt{3}}\right]M_D$$

$$\cong 2.155M_D$$

And thus we find that the airframe must be at least 2.155 times the diameter of the motor mount tubes we plan to use for our 3-engine clustered rocket.

Let's take an example: suppose we want to build a 3-engine cluster using 18 mm motors. We know that we'll have to use BT-20 tubing for the motor mount tubes (outside diameter = 0.736"), so AF_D= 2.155 x 0.736" = 1.586". If we expect to use a commercially available airframe for the rocket, then we know that we'll have to use a BT-60 (inner diameter = 1.595") to accommodate this cluster. This is in fact the case as evidenced, for example, by the old Estes Ranger kit, K-6.

4.0 The 4-Engine Cluster

This case examines four engines arranged in a square, as shown in Figure 4 below:

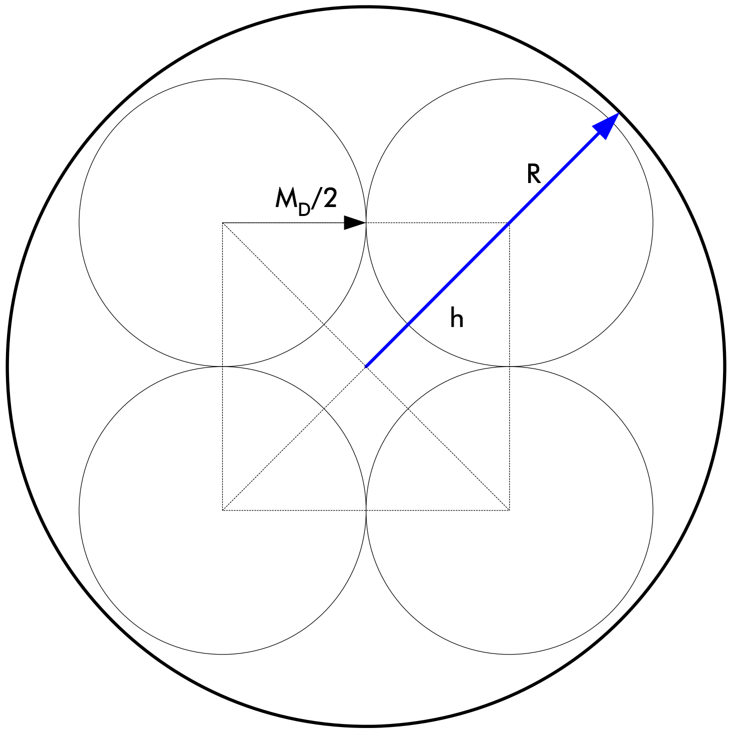

Figure 4: 4-Engine Cluster

The Airframe radius is represented by the blue line, R. It is evident that:

$R = h + \frac{M_D}{2}$; h being the distance from the airframe center to the vertex of the square.

$\therefore AF_D \geq 2h + M_D$

Using the Pythagorean Theorem, we can declare that:

$$h^2 + h^2 = 2h^2 = M_D^2$$

$$\therefore h = \frac{M_D}{\sqrt{2}}$$

Substituting this result into our expression for AF_D we get:

$$AF_D \geq \frac{2M_D}{\sqrt{2}} + M_D$$

$$\therefore AF_D \geq \left(1 + \sqrt{2}\right)M_D \cong 2.414M_D$$

5.0 Generalizing the Formula

The foregoing analysis seems like a lot of work when one can just pick up a few tubes and dry-fit them together. For example, if all one was interested in was just a 3-engine cluster then the math wouldn't really be necessary. Where the math does help is in those cases where we might wish to make a more complex cluster. Also, as we increase the size of the cluster arrangement, a central void between the motor mount tubes appears, and we might be interested in calculating the maximum motor size that could fit in this center position.

However it is clear that analyzing the more complex cluster arrangements with just the Pythagorean Theorem soon renders the process unwieldy. With the aid of some elementary trigonometry, the following sections develop a general solution for any n-sided, regular polygonal, engine cluster arrangement.

5.1 The General Solution

The assumption is made that the Builder intends to arrange the engines in a regular polygonal arrangement. For implementation efficiency we will further assume that the engines are all of equal diameter and are just touching each other, which in turn means that each engine has its respective center point located at a vertex of the polygon.

Practically speaking, this means that a 4-engine cluster would have the engines arranged in a square; a 5-engine cluster would be arranged in a pentagon; a 6-engine cluster would be arranged in a hexagon, and so on. What also becomes interesting is the size of the central void that appears between the engines. This central void grows as the size of the cluster grows, and presents an opportunity to add additional engines to the cluster.

To examine the general case, we must first recall the characteristics of Regular Polygons. Regular Polygons are defined as closed figures possessing sides of equal length. It can then be shown that the magnitude of each interior angle is directly related to the number of sides, as follows:

$|\angle| = \frac{180°(n-2)}{n}$; where n= the number of sides of the Regular Polygon.

The proof for this is provided in the Appendix to this paper.

Let's now apply this information to the general problem. Figure 5 illustrates the geometry for a complex cluster arranged in an n-sided, regular polygonal layout. For the sake of clarity, only a portion of the cluster is shown:

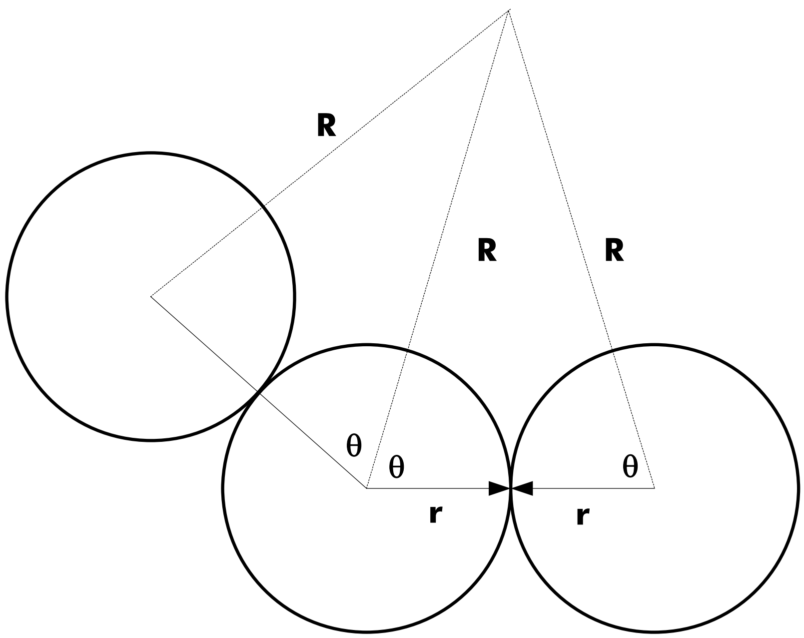

Figure 5: Complex Cluster of 'n' Engines

From this figure we can see that the interior angle at any vertex is 2θ. The minimum radius of the airframe that will just accommodate this cluster will be equal to the distance from the center point to a vertex, plus the radius of the motor mount tube. We can express these relationships as follows:

$$2\theta = \frac{180°(n-2)}{n}\ ;\ \therefore \theta = \frac{90°(n-2)}{n}$$

And

$$\frac{AF_D}{2} \geq R + r\ ;\ \therefore AF_D \geq 2(R + r)$$

From the triangle we find the following relationship:

$$\frac{r}{R} = \cos\theta\ ;\ \therefore R = \frac{r}{\cos\theta}$$

$$\therefore AF_D \geq 2\left[r + \frac{r}{\cos\theta}\right] = 2r\left[1 + \frac{1}{\cos\theta}\right]$$

But $r = \frac{M_D}{2}$ and $\theta = \frac{90°(n-2)}{n}$

So $AF_D \geq M_D \left[1 + \frac{1}{\cos\left[\frac{90°(n-2)}{n}\right]} \right]$

This can be further simplified by taking advantage of the basic trigonometric identity $\cos\theta = \sin(90° - \theta)$. With this, and a bit more algebra, we get:

$$AF_D \geq M_D \left[1 + \frac{1}{\sin\left(\frac{180°}{n}\right)} \right];\ n \geq 2$$

This result gives us a simple expression that defines the minimum inside diameter of the airframe needed to accommodate a cluster of n engines.

5.2 The Central Void

As the size of the cluster is increased, the diameter of the middle space around which the engines are arranged also increases. For small clusters of small engines (for example, a cluster of three 13 mm engines) this middle space, or Central Void, is very small and not that useful. But in larger clusters of larger engines, the Central Void offers a space that could be used to include additional engines. We can derive an expression for the Central Void that will permit us to calculate the diameter of this space.

Let's start by illustrating the problem as shown below in Figure 6. In the figure we have a large cluster of engines arranged at the vertices of a high order (large 'n') polygon. For the sake of clarity, most of the motor mount tubes have been removed so we can focus on the key parameters.

Let's define the diameter of the Central Void as CV_D.

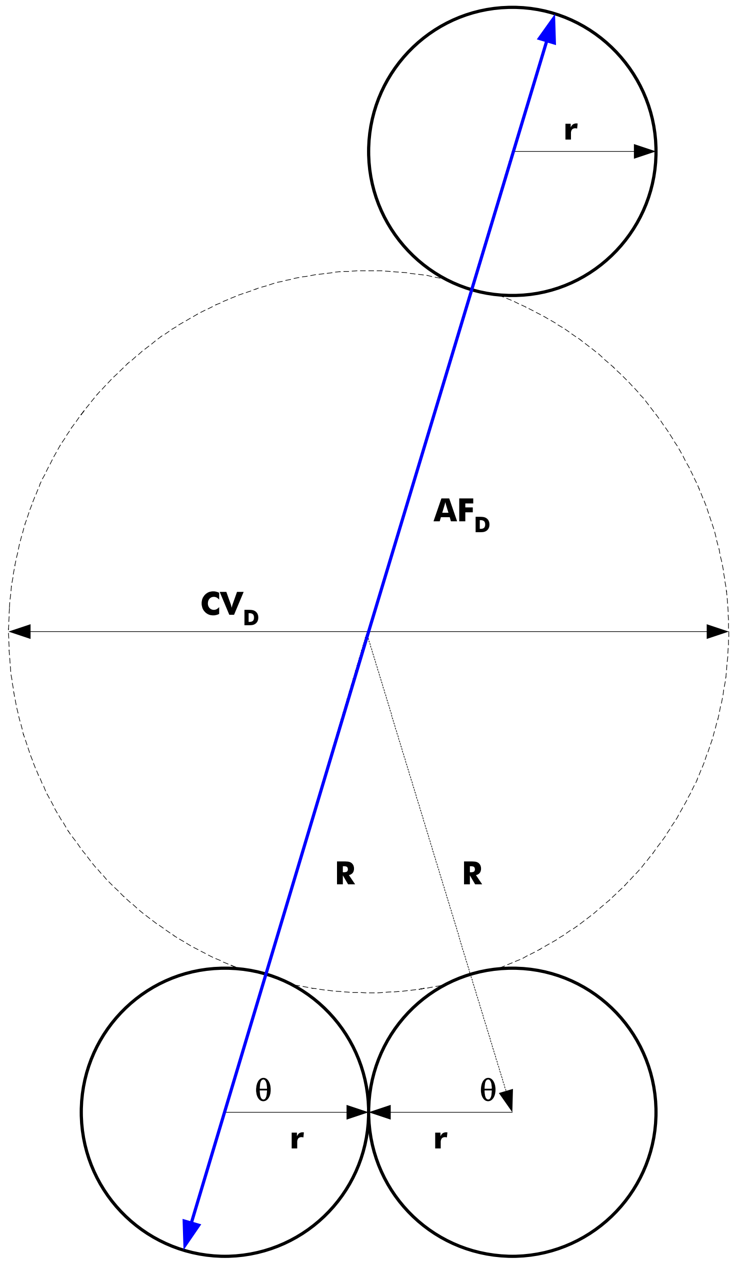

Figure 6: The Central Void

From Figure 6 it is readily apparent that the inside diameter of the Airframe is the sum of CV_D plus two motor mount tubes. This is represented in the figure by the blue line, AF_D, and can be expressed as follows:

$$AF_D \geq 2M_D + CV_D$$

So it follows that:

$$CV_D \le AF_D - 2M_D$$

$$\text{But } AF_D \ge M_D\left[1 + \frac{1}{\sin\left(\frac{180°}{n}\right)}\right]$$

$$\text{So } CV_D \le M_D\left[1 + \frac{1}{\sin\left(\frac{180°}{n}\right)}\right] - 2M_D$$

$$\le M_D + \frac{M_D}{\sin\left(\frac{180°}{n}\right)} - 2M_D$$

$$\le M_D\left[\frac{1}{\sin\left(\frac{180°}{n}\right)} - 1\right];\ n \ge 2$$

The following table summarizes the equations to calculate the airframe diameter and the size of the Central Void for various cluster sizes. It is evident from the table that the Central Void diameter is precisely 2M less than the Airframe diameter, just as the initial expression suggested.

Table 1: Polygonal Cluster Summary

# Of Engines	Cluster Arrangement	$AF_D \ge$	$CV_D \le$
2	Pair	2 x M_D	No Void
3	Triangle	2.155 x M_D	0.155 x M_D
4	Square	2.414 x M_D	0.414 x M_D
5	Pentagon	2.701 x M_D	0.701 x M_D
6	Hexagon	3 x M_D	1 x M_D
7	Heptagon	3.305 x M_D	1.305 x M_D
8	Octagon	3.613 x M_D	1.613 x M_D

Of particular interest in the table is the entry for the hexagonal cluster. There, we can see that the Central Void space is precisely the same size as the motor mount tubes used for the cluster. This also means that six tubes of identical diameter will fit precisely around a central tube of the same diameter. This property of the hexagonal arrangement is well known, as it is commonly evidenced in tube-stabilized rockets, such as The Squirrel Works Tuber, and others.

5.4 Finding 'n'

Suppose you have an airframe in hand and simply want to find out how many engines of a given size will fit within it? This is easily found by re-arranging the expression for AF_D to solve for n, as follows:

$$AF_D \geq M_D \left[1 + \frac{1}{\sin\left(\frac{180^\circ}{n}\right)} \right]$$

$$\frac{AF_D}{M_D} \geq 1 + \frac{1}{\sin\left(\frac{180^\circ}{n}\right)}$$

$$\frac{AF_D}{M_D} - 1 \geq \frac{1}{\sin\left(\frac{180^\circ}{n}\right)}$$

$$\frac{AF_D - M_D}{M_D} \geq \frac{1}{\sin\left(\frac{180^\circ}{n}\right)}$$

Inverting and re-arranging gives:

$$\sin\left(\frac{180^\circ}{n}\right) \geq \frac{M_D}{AF_D - M_D}$$

$$\frac{180^\circ}{n} \geq \arcsin\left[\frac{M_D}{AF_D - M_D}\right]$$

And thus:

$$n \leq \frac{180^\circ}{\arcsin\left[\frac{M_D}{AF_D - M_D}\right]} \quad ; AF_D > M_D$$

Since 'n' represents the number of engines its value has to be an integer, so the calculated result has to be rounded down to the nearest whole number for practical applications. We express this as follows:

$$n = \mathrm{INT}\left\{ \frac{180^\circ}{\arcsin\left[\frac{M_D}{AF_D - M_D}\right]} \right\} \quad ; AF_D > M_D$$

5.4 Making Use of the Central Void

We now have the tools at our disposal to readily calculate the engine combinations for a complex cluster. The best way to illustrate this is with an example.

Let's say we wish to build a 4" rocket and we would like to maximize the cluster of engines. The airframe we will use is a LOC-3.9 and we wish to use 24 mm engines for the outer ring of the cluster. The LOC-3.9 airframe has an ID of 3.90" and we will use the metal foil lined, heavy walled 24 mm motor mount tubes from BMS (T50MF) – these have an outside diameter of 1.000".

With these parameters, and using the formula for n, we calculate that the maximum cluster that will fit within this airframe is eight 24 mm engines, to be arranged in the shape of an octagon.

Using the formula for CV_D we calculate that this cluster will have a Central Void of 1.613". This void would accommodate one 38 mm engine (using LOC-1.52 tubing), or if we use the equation for n again, we find that either a cluster of 3 BT-20 tubes or 5 BT-5 tubes would also fit within that center space.

Part II:

Complex Clusters

1.0 Introduction

In most applications, modelers will find that a simple polygonal cluster will meet their needs. In Part I, a set of general expressions were found that can be used to calculate the size of the components needed to realize a particular polygonal cluster, and these results can be used to select the right commercial components for the design.

However, a modeler might be interested in experimenting with more complex cluster arrangements, or perhaps there might be an interest in determining the maximum number of engines (in various combinations) that might fit into a given airframe. This Part II explores several of the fundamental complex cluster arrangements with the intention of developing a set of basic analytical tools that will aid the modeler in the design of his own complex cluster.

1.1 Definition of Terms

AF_D: The minimum inside diameter of an airframe that will just accommodate the desired motor cluster.

Interference Fit: The term used to denote the condition that occurs when a secondary motor mount tube is inserted in the gap that exists between the outside of the cluster and the inside of the airframe, the secondary motor mount being just tangent to these components. This secondary motor mount tube is designated "M_2" in this paper.

M_1: The outside diameter of Motor Mount Tube 1.

M_2: The outside diameter of Motor Mount Tube 2.

'n': The number of sides in an n-sided polygon; also the number of engines in a cluster.

Regular Polygon: A closed, 2-dimensional figure possessing sides of equal length and interior angles of equal value.

1.2 Summary of Results

Equilateral Clusters:

i) Section 2.1: 2-Engine Equilateral Layout

$$AF_D \geq M_2 + \frac{\sqrt{3}M_1}{3} + \sqrt{M_2^2 + 2M_1M_2}$$

Interference Fit: $M_2 \leq \frac{(2+\sqrt{3})}{(6+\sqrt{3})}M_1 \cong 0.483M_1$

ii) Section 2.3: Full Equilateral Layout

$$AF_D \geq \left(\frac{3+4\sqrt{3}}{3}\right)M \cong 3.309M$$

iii) Section 2.4: 2-Engine Full Equilateral

$$AF_D \geq M_2 + \sqrt{M_1^2 + \frac{1}{3}\left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2}$$

Interference Fit: $M_2 \leq 0.649M_1$

Square Clusters:

i) Section 3.1: Complex Square Cluster

$$AF_D \geq M_1 + M_2 + \sqrt{M_2^2 + 2M_1M_2}$$

Interference Fit: $M_2 \leq \frac{M_1}{\left(1+\sqrt{2}\right)} \cong 0.414M_1$

Rhombic Clusters:

i) Section 4.1: 2-Engine Rhombus

$$AF_D \geq M_2 + \sqrt{M_2^2 + 2M_1M_2}$$

Interference Fit: $M_2 \leq \frac{2M_1}{3}$

ii) Section 4.3: The Full Rhombus

$$AF_D \geq \left(1+\sqrt{3}\right)M \cong 2.732M$$

iii) Section 4.4: The 2-Engine Full Rhombus

$$AF_D \geq M_2 + \sqrt{\left(M_1 + M_2\right)^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}}$$

Interference Fit: $M_2 \leq 0.581M_1$

2.0 Equilateral Clusters

In Part I, Section 3.0, the basic 3-engine cluster was examined. In this case, we will introduce a second engine, M_2, into the space adjacent to the cluster and the inner airframe wall, as illustrated in Figure 7, below.

A critical observation we make concerns M_2, when its size (diameter) is permitted to grow to fit just tangent within the outer cavity in the cluster (refer to Figure 7, below). This tangent diameter is defined as the Interference Fit; for a given airframe and main cluster (M_1), M_2 must have a diameter just equal to or less than the size of this cavity or otherwise the engine will not fit into this space. We will use this tangent case later on to find a definitive expression that relates the diameter of M_2 to M_1.

When M_2 is less than or equal to the Interference Fit, the minimum inner airframe diameter, AF_D, is governed by the equation for the main engine cluster. If this cluster layout is a regular polygon, then the relationships found in Part I apply. However, once M_2 exceeds the Interference Fit diameter, the airframe obviously must expand to accommodate the larger mixed motor configuration. The equation for AF_D must then take this expanded geometry into account. Here, in Part II, the analysis considers the impact caused by introducing a second engine into several standard cluster layouts, and develops formulas for finding the minimum inner airframe diameter in this event.

2.1 The 2-Engine Equilateral

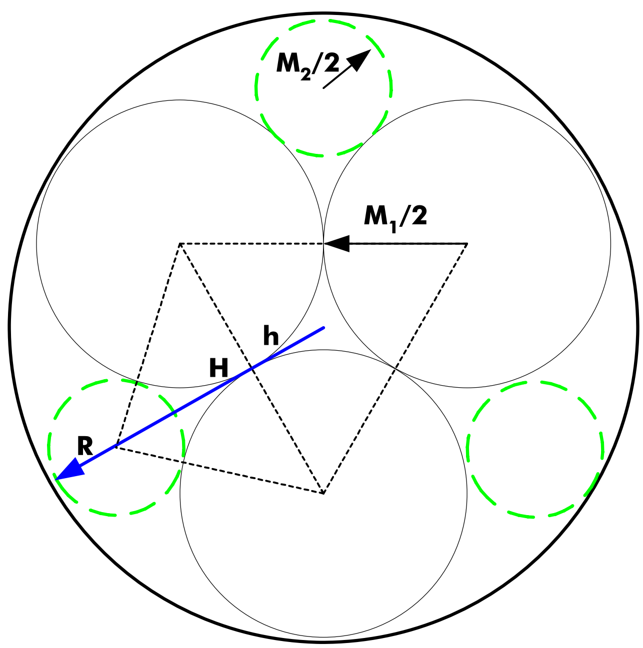

Figure 7: 2-Engine Equilateral Layout

To begin, we declare the following definitions:

- h = the distance from the midpoint of any side of the equilateral triangle to its center.

- H = the height of the isosceles triangle (base to center of M_2)
- R = Airframe radius, as represented by the blue line.
- AF_D = 2R.

From the diagram, we can see that $R = h + H + \frac{M_2}{2}$.

$\therefore AF_D \geq 2(h + H) + M_2$

In Part I, Section 3.0, we found that $h = \frac{M_1}{2\sqrt{3}}$; we'll re-use that fact here. This just leaves finding the expression for H to complete the formula for the airframe diameter.

Considering the isosceles triangle, we see that:

$$H^2 + \left(\frac{M_1^2}{2}\right)^2 = \left(\frac{M_1 + M_2}{2}\right)^2$$

$$4H^2 = M_2^2 + 2M_1M_2$$

$$\therefore H^2 = \frac{1}{4}\left(M_2^2 + 2M_1M_2\right)$$

And $H = \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2}$

So $AF_D \geq 2\left(\frac{M_1}{2\sqrt{3}} + \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2}\right) + M_2$

Or $AF_D \geq M_2 + \frac{\sqrt{3}M_1}{3} + \sqrt{M_2^2 + 2M_1M_2}$

This expression gives us the minimum airframe diameter needed to house an equilateral cluster layout consisting of any two engine sizes, M_1 and M_2.

2.2 Interference Fit

In the absence of M_2, the minimum airframe diameter will be established by the formula for a 3-engine cluster, as found in Part I, Section 3.0:

$$AF_D \geq \left(\frac{3 + 2\sqrt{3}}{3}\right)M_1$$

However, when M_2 is introduced and its diameter is permitted to grow, then interference occurs when M_2 becomes tangent to the airframe and the cluster. At this point, we have the following equality:

$$\left(\frac{3+2\sqrt{3}}{3}\right)M_1 = M_2 + \frac{\sqrt{3}M_1}{3} + \sqrt{M_2^2 + 2M_1M_2}$$

We now solve for M_2, thus:

$$\left(3+2\sqrt{3}\right)M_1 = 3M_2 + \sqrt{3}M_1 + 3\sqrt{M_2^2 + 2M_1M_2}$$

$$\left(3+2\sqrt{3}\right)M_1 - 3M_2 = \sqrt{9M_2^2 + 18M_1M_2}$$

$$\left[\left(3+2\sqrt{3}\right)M_1 - 3M_2\right]^2 = 9M_2^2 + 18M_1M_2$$

$$\left(12+6\sqrt{3}\right)M_1^2 - \left(18+6\sqrt{3}\right)M_1M_2 + 9M_2^2 = 9M_2^2 + 18M_1M_2$$

$$\left(12+6\sqrt{3}\right)M_1 = \left(36+6\sqrt{3}\right)M_2$$

$$\therefore M_2 \le \frac{\left(2+\sqrt{3}\right)}{\left(6+\sqrt{3}\right)}M_1 \cong 0.483M_1$$

The result indicates that M_2 must be equal to or less than $0.483M_1$ if it is to fit in the adjacent space in a minimum 3-engine cluster airframe.

For example, if our main engines were 24 mm, the motor mounts would be at least BT-50 size (0.976”). Using the various equations, we would find that a cluster of three 24 mm engines would just fit within a BT-70, but the Interference Fit equation confirms that we would not be able to fit a 13 mm motor (BT-5 motor mount, at 0.544” diameter) in the adjacent spaces. For this combination (three 24 mm engines and three 13 mm engines) to work, we would need to use either a BT-80 or a LOC 2.56 airframe for the rocket.

2.3 Full Equilateral Layout

If the diameter of M_2 is increased beyond the Interference Fit limits in Figure 7, then the diameter of the airframe will need to increase. We now consider the case when M_2 reaches the same diameter as M_1; the cluster configuration begins to take the shape of a honeycomb, as illustrated in the following figure:

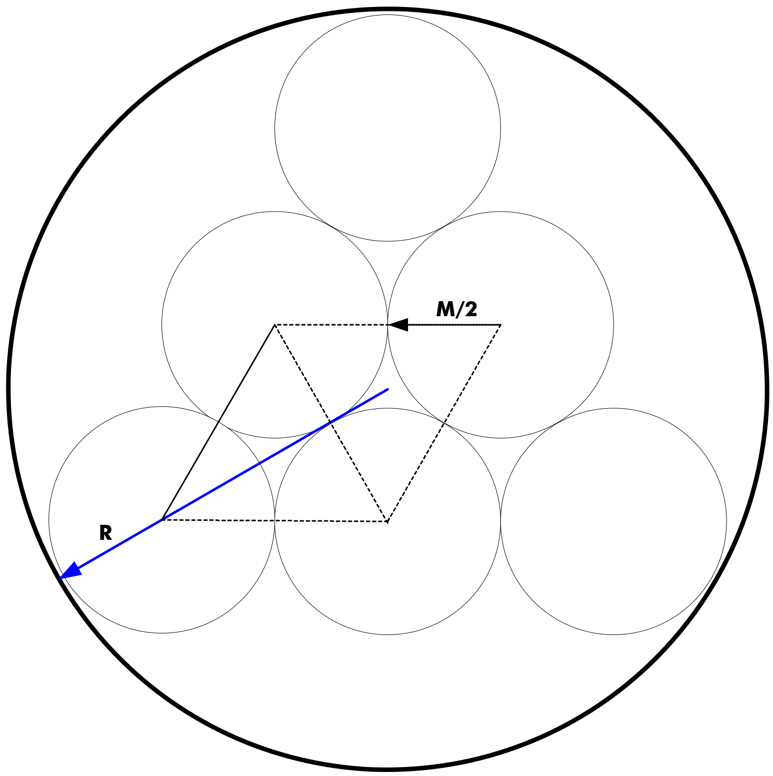

Figure 8: Full Equilateral Layout

Mathematically, we will re-use our findings from Section 2.1, above. Since in this case we have set $M_1 = M_2$, we can simply set the engine variables in the equation to the same value – let's call this M.

Thus: $$AF_D \geq M + \frac{\sqrt{3}M}{3} + \sqrt{M^2 + 2M^2}$$

Or: $$AF_D \geq \left(\frac{3 + 4\sqrt{3}}{3} \right) M \cong 3.309M$$

We can continue to build out the equilateral arrangement, which will of course require larger and larger airframes to house the configuration. Mathematically, an arithmetic progression begins to reveal itself in the equation for AF_D, but we'll save the analysis of this progression for another day.

2.4 The 2-Engine Full Equilateral

In this larger equilateral configuration, more space opens up to introduce a second engine type into the outer cavities of the airframe, as illustrated in Figure 9 below:

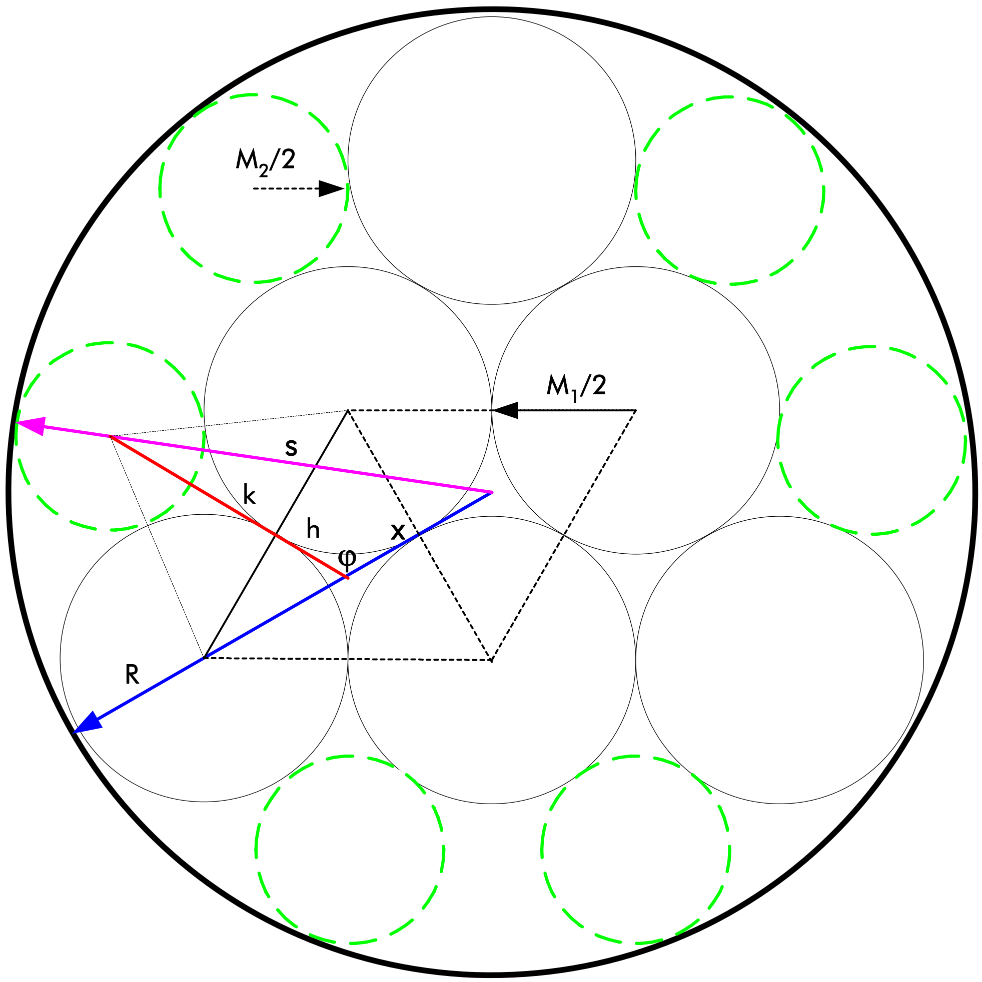

Figure 9: 2-Engine Full Equilateral

We begin this analysis by observing that M_2 sits on a radial line (the pink line) that is offset from its tangent line (the red line, comprising segments h and k). In each of the previous complex cluster examples, these two lines were coincident, a feature that greatly simplified the analysis. In this case, the offset complicates the solution for AF_D, precipitating a quadratic that must be solved to find the Interference Fit. Later in section 4.4, we will see the same effect occur in the solution for the complex rhombic layout.

Accordingly, we declare the following definitions:

- h = the distance from the midpoint of the equilateral triangle to a side; we recall from Part I, Section 3.0, that $h = \frac{M_1}{2\sqrt{3}}$ for an equilateral.

- **k** = the height of the isosceles triangle. From Part II, Section 2.1, we recall that $k = \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2}$

- **Q** = h + k, represented in the figure by the red line.

- **x** = the distance from the center of the cluster to the lower end of Q. It is evident from the figure that $x = 2h = \frac{M_1}{\sqrt{3}}$

- **s** = the distance from the center of the airframe to the center of M_2.

- **φ** = 120°. This is evident, as **φ** is the angle formed by the side bisectors that converge in the middle of the equilateral triangle.

- **AF_D** = 2R, and is represented by the blue line in the diagram.

With the Law of Cosines, we can establish the following equality:

$$s^2 = x^2 + Q^2 - 2xQ\cos(120°)$$

But $\cos(120°) = -\frac{1}{2}$

$$\therefore s^2 = x^2 + Q^2 + xQ$$

Now $Q = h + k$

$$\therefore Q = \frac{M_1}{2\sqrt{3}} + \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2} = \frac{1}{2\sqrt{3}}\left[M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right]$$

Processing the components for s^2, we get:

$$s^2 = \frac{7M_1^2}{12} + \frac{M_1}{3}\sqrt{3M_2^2 + 6M_1M_2} + \frac{M_1M_2}{2} + \frac{M_2^2}{4}$$

$$s^2 = \frac{1}{12}\left[7M_1^2 + 4M_1\sqrt{3M_2^2 + 6M_1M_2} + 6M_1M_2 + 3M_2^2\right]$$

And with some final factoring we get:

$$s^2 = \frac{1}{12}\left[3M_1^2 + \left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2\right]$$

Taking the square root gives us s:

$$s = \frac{1}{2\sqrt{3}}\sqrt{3M_1^2 + \left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2}$$

$$\therefore s = \frac{1}{2}\sqrt{M_1^2 + \frac{1}{3}\left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2}$$

This result now permits us to find AF_D. Observing the pink line in the figure, it is evident that $AF_D = 2s + M_2$;

$$\therefore AF_D \geq M_2 + \sqrt{M_1^2 + \frac{1}{3}\left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2}$$

2.5 Interference Fit:

Section 2.3 found the minimum airframe diameter for a full equilateral cluster to be

$$AF_D \geq \left(\frac{3 + 4\sqrt{3}}{3}\right)M$$

Interference occurs when a second engine, M_2, is introduced into the adjacent spaces and its diameter is tangent with the primary engines and the inner airframe wall. Doing so here creates the following equality:

$$\frac{\left(3 + 4\sqrt{3}\right)M_1}{3} = M_2 + \sqrt{M_1^2 + \frac{1}{3}\left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2}$$

$$\left[\left(\frac{3 + 4\sqrt{3}}{3}\right)M_1 - M_2\right]^2 = M_1^2 + \frac{1}{3}\left(2M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)^2$$

Expanding the relation we get:

$$\frac{\left(57 + 24\sqrt{3}\right)}{3}M_1^2 - \left(6 + 8\sqrt{3}\right)M_1M_2 + 3M_2^2 = 7M_1^2 + 4M_1\sqrt{3M_2^2 + 6M_1M_2} + 6M_1M_2 + 3M_2^2$$

$$12M_1\sqrt{3M_2^2 + 6M_1M_2} = \left(36 + 24\sqrt{3}\right)M_1^2 - \left(36 + 24\sqrt{3}\right)M_1M_2$$

Dividing by $12M_1$, we get:

$$\sqrt{3M_2^2 + 6M_1M_2} = \left(3 + 2\sqrt{3}\right)\left(M_1 - M_2\right)$$

Squaring and then dividing by 3 gives us:

$$M_2^2 + 2M_1M_2 = \left(7 + 4\sqrt{3}\right)M_1^2 - \left(14 + 8\sqrt{3}\right)M_1M_2 + \left(7 + 4\sqrt{3}\right)M_2^2$$

This leaves us with a quadratic to solve for M_2 as follows:

$$\left(6 + 4\sqrt{3}\right)M_2^2 - \left(16 + 8\sqrt{3}\right)M_1M_2 + \left(7 + 4\sqrt{3}\right)M_1^2 = 0$$

The roots are:

$$M_2 = \frac{\left(8 + 4\sqrt{3}\right)M_1 \pm \sqrt{\left(22 + 12\sqrt{3}\right)M_1^2}}{\left(6 + 4\sqrt{3}\right)}$$

Or $M_2 \cong \dfrac{14.928M_1 \pm 6.541M_1}{12.928}$

$$\therefore M_2 = \left(1.155 \pm 0.506\right)M_1$$

Since M_2 must be less than M_1 at the Interference Fit point, we must take the smaller root.

Therefore $M_2 \leq 0.649M_1$

3.0 Square Clusters

3.1 Complex Square Cluster

In Part I, Section 4.0, the 4-engine cluster was examined. In this example, a second engine, M_2, is introduced into the cluster. Figure 9 illustrates this configuration:

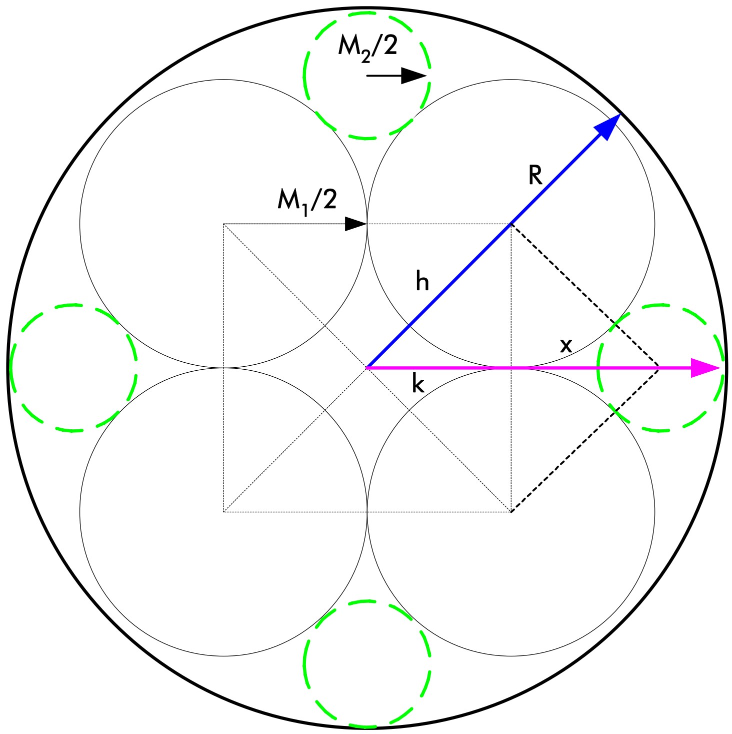

Figure 10: Complex Square Cluster

We declare the following definitions:

- $h = \frac{M_1}{\sqrt{2}}$, as found in Part I, Section 4.0.
- k = the distance from the center of the square to the midpoint of any side.
- x = the height of the isosceles triangle.
- R = the radius of the airframe, as equally represented by the blue and pink lines.

Examining the pink line, we see that:

$$R = k + x + \frac{M_2}{2}$$

But $AF_D = 2R$

So $AF_D = 2(k + x) + M_2$

From the diagram we can see that:

$$k^2 + \left(\frac{M_1}{2}\right)^2 = \left(\frac{M_1}{\sqrt{2}}\right)^2$$

$$\text{So } k^2 = \frac{M_1^2}{2} - \frac{M_1^2}{4} = \frac{M_1^2}{4}$$

$$\therefore k = \frac{M_1}{2}$$

We can also see that:

$$x^2 + \left(\frac{M_1}{2}\right)^2 = \left(\frac{M_1 + M_2}{2}\right)^2$$

$$4x^2 = M_2^2 + 2M_1M_2$$

$$\therefore x = \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2}$$

Substituting these expressions for k and x into the equation for AF_D, we get:

$$AF_D \geq M_1 + M_2 + \sqrt{M_2^2 + 2M_1M_2}$$

3.2 Interference Fit

Interference occurs when M_2 is just tangent to the cluster and the airframe wall. In this event we have the following equality:

$$\left(1 + \sqrt{2}\right)M_1 = M_1 + M_2 + \sqrt{M_2^2 + 2M_1M_2}$$

$$\sqrt{2}M_1 - M_2 = \sqrt{M_2^2 + 2M_1M_2}$$

$$\left(\sqrt{2}M_1 - M_2\right)^2 = M_2^2 + 2M_1M_2$$

$$M_1 - \sqrt{2}M_2 = M_2$$

$$\therefore M_2 \leq \frac{M_1}{\left(1 + \sqrt{2}\right)} \cong 0.414M_1$$

Of interest in this case is the fact that the Interference Fit size for M_2 is the same size as the Central Void; see the entry for a Square in Table 1, Part 1, Section 5.2.

3.3 Setting M_2 to M_1

Referring to Figure 10, if the diameter for M_2 is permitted to grow to the same size as M_1, we arrive at an eight-engine cluster, similar to the engine layout found in the Saturn 1b (although the four outer engines in the Saturn 1b are not tangent to the inner four).

In this case, we can re-use the equation for AF_D from Section 3.1 and set $M_1 = M_2 = M$. Accordingly, the equation reduces as follows:

$$AF_D \geq 2M + \sqrt{M^2 + 2M^2}$$

$$\therefore AF_D = \left(2 + \sqrt{3}\right)M$$

For example, if M was selected to be a BT-5, then an eight-engine cluster of this configuration would fit just nicely inside a BT-70.

4.0 Rhombic Clusters

4.1 The 2-Engine Rhombic Cluster

Back in Part 1, Section 2.0, we examined the very simple 2-engine cluster. It was readily apparent in that simple example that the minimum airframe diameter for this cluster is $AF_D \geq 2M_1$. The following figure now introduces a second engine, M_2, into the empty space adjacent to the cluster.

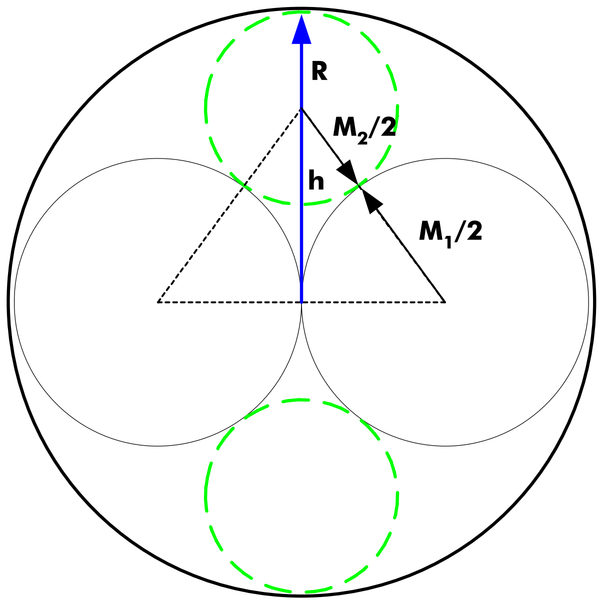

Figure 11: 2-Engine Rhombic Cluster

To begin, we declare the following definitions:

- The minimum airframe radius accommodating this cluster is **R**, as represented by the blue line in the diagram. Therefore the minimum airframe diameter is given by $AF_D = 2R$.
- h = the height of the isosceles triangle formed by the vertices of M_1 and M_2.

From the diagram, it can be seen that:

$$R = h + \frac{M_2}{2};$$

$$\therefore AF_D \geq 2h + M_2$$

But $h^2 + \left(\frac{M_1}{2}\right)^2 = \left(\frac{M_1 + M_2}{2}\right)^2$

So $4h^2 + M_1^2 = M_1^2 + 2M_1M_2 + M_2^2$

$$\therefore h^2 = \frac{1}{4}\left(M_2^2 + 2M_1M_2\right)$$

And $h = \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2}$

Substituting this result into our expression for AF_D, we get:

$$AF_D \geq M_2 + \sqrt{M_2^2 + 2M_1M_2}$$

This expression finds the minimum airframe diameter needed to accommodate a rhombic cluster comprised of two known engine diameters.

4.2 Finding the Interference Fit for M_2

Interference occurs when M_2 is just tangent to the cluster and the inner airframe wall. Thus we have the following equality:

$$2M_1 = M_2 + \sqrt{M_2^2 + 2M_1M_2}$$

$$(2M_1 - M_2)^2 = M_2^2 + 2M_1M_2$$

$$4M_1^2 - 4M_1M_2 + M_2^2 = M_2^2 + 2M_1M_2$$

$$4M_1^2 = 6M_1M_2$$

$$2M_1 = 3M_2$$

$$\therefore M_2 \leq \frac{2M_1}{3}$$

Let's take an example: let's say that we wish to build a 2-engine cluster using 24 mm engines. This means that the motor mount tubes must be at least BT-50 size and therefore AF_D must be greater than 2 x 0.976" = 1.952". The closest commercial airframe would be ST-20, with an ID = 2.00".

We might imagine that an 18 mm engine could fit in the adjacent cavities, creating a 4-engine rhombic cluster. Using the airframe diameter formula for this configuration, we find that AF_D must be at least 2.143", clearly indicating this dual engine configuration will not fit in the ST-20 (although it would fit in a BT-70).

Knowing that if the main motor mount tubes are 0.976" and that this pair just fits inside the ST-20, we can use the Interference Fit expression to find M_2. This gives us an M_2 less than 0.651", also making it clear that a rhombic cluster of two 24 mm engines and two 18 mm engines will not fit inside the ST-20. With a pair of 24 mm engines as the main cluster, we must settle for two 13 mm secondary engines if we are to use an ST-20 airframe. Bumping the airframe up to a BT-70 enables the 24 mm/18 mm combination.

Alternatively, if the main engine pair consists of two 29 mm engines (LOC 29 mm motor mount tubes), then we find that this combination will fit in either a BT-80 or LOC 2.56 airframe, and that the largest secondary engines will be 18 mm (BT-20 motor mount tubes).

4.3 The Full Rhombus

If the diameter of M_2 is permitted to grow to the same size as M_1, we arrive at a full rhombus configuration. The following figure illustrates this arrangement:

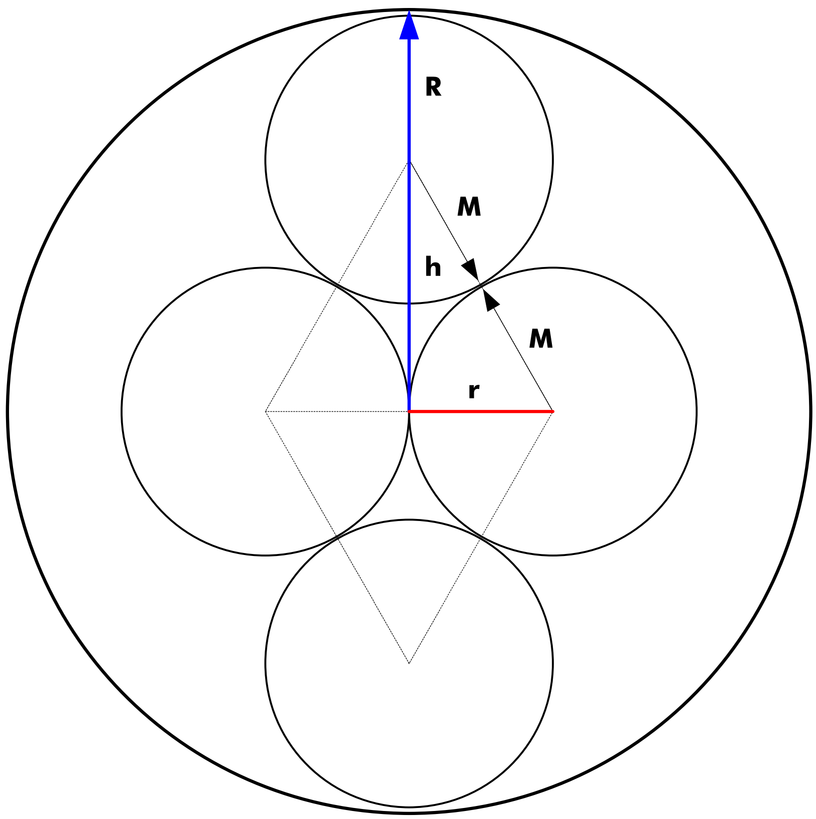

Figure 12: Full Rhombus Cluster

We declare the following definitions:

- h = the distance from the center of the airframe to the center of the outer engine (i.e.: the height of the equilateral triangle in the figure).
- R = the radius of the airframe, as represented by the blue line.

From the diagram we can see that:

$$R = h + \frac{M}{2}$$

But $AF_D = 2R$

$$\therefore AF_D \geq 2h + M$$

We recall that for equilateral triangles $h = \frac{\sqrt{3}M}{2}$

$$\therefore AF_D \geq \left(1+\sqrt{3}\right)M \cong 2.732M$$

Note that we arrive at exactly the same result by setting $M_2 = M_1 = M$ in the expression for AF_D found in Section 4.1.

4.4 The 2-Engine Full Rhombus

If we introduce a second engine into this arrangement, we arrive at the following configuration:

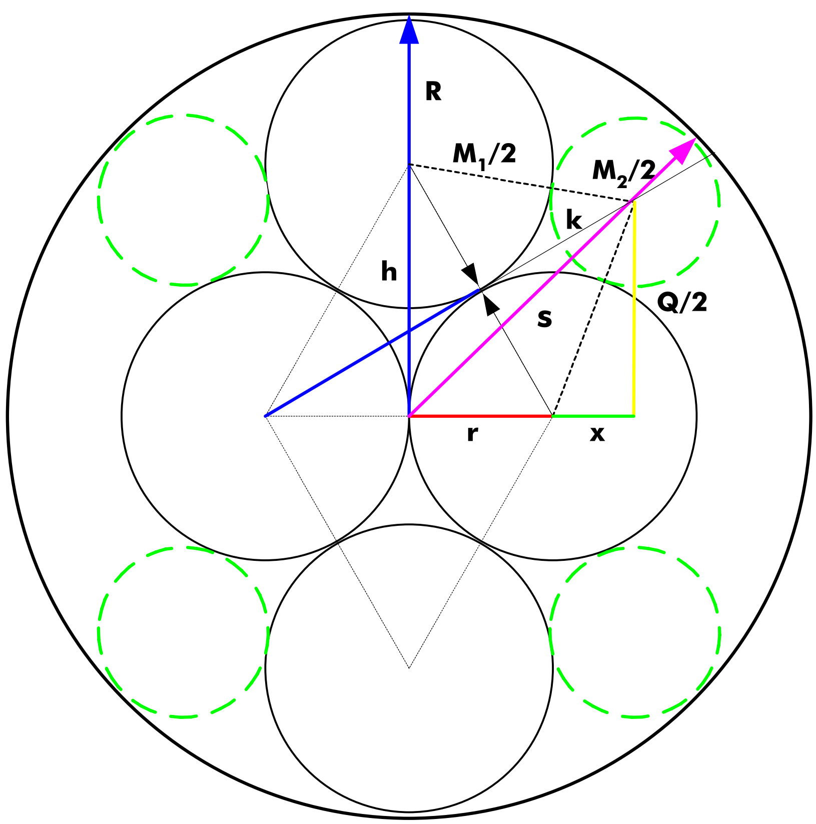

Figure 13: 2-Engine Full Rhombus

We begin this analysis by declaring the following definitions:

- **h** = the height of the internal equilateral triangle. Here, $h = \frac{\sqrt{3}M_1}{2}$.
- **k** = the height of the isosceles triangle formed by the radial components of M_1 and M_2.
- **Q** = h + k, and represents the distance along the tangent line from the base vertex of the equilateral triangle to the center of M_2.
- $r = \frac{M_1}{2}$, the radius of M_1.
- **s** = the distance along the radial line from the center of the configuration to the center point of M_2.
- **R** = the minimum radius of the airframe and is represented by the blue line. As usual, AF_D = 2R. As shown in Part II, Section 4.3, $AF_D = (1+\sqrt{3})M_1$ so long as $M_2 < M_1$. This result will be used to formulate the Interference Fit.
- **x** = the remaining segment necessary to complete the larger right triangle formed by the centers of the central M_1 motor mount tubes and the center of M_2. It will be shown that this right triangle is half of an equilateral, and this fact will be central to the analysis.

To find the minimum airframe diameter that will accommodate both M_1 and M_2 in this mixed configuration, we will need to characterize a radial line that passes through these two motor mounts. The pink line does this, and one can readily see that:

$$AF_D \geq 2\left(s + \frac{M_2}{2}\right)$$

$$\therefore AF_D \geq 2s + M_2$$

We now need to find an expression for s involving both M_1 and M_2. To do this, we will need to characterize the segments that comprise the right triangle. Thus:

$$h = \frac{\sqrt{3}M_1}{2}$$

And we can see that:

$$k^2 + \left(\frac{M_1}{2}\right)^2 = \left(\frac{M_1 + M_2}{2}\right)^2$$

$$\therefore 4k^2 = M_2^2 + 2M_1M_2$$

$$\therefore k = \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2}$$

Now $Q = h + k$;

$$\therefore Q = \frac{\sqrt{3}M_1}{2} + \frac{1}{2}\sqrt{M_2^2 + 2M_1M_2} = \frac{1}{2}\left(\sqrt{3}M_1 + \sqrt{M_2^2 + 2M_1M_2}\right)$$

Inspecting the diagram, it becomes apparent that Q is a line that bisects the lower left vertex of the equilateral triangle and is the hypotenuse of the larger right triangle. This means the larger right triangle must be half of a large equilateral, whose height is $2r + x$. We therefore can declare the following:

$$2r + x = M_1 + x = \frac{\sqrt{3}Q}{2}$$

$$\therefore x = \frac{\sqrt{3}Q}{2} - M_1 = \frac{\sqrt{3}}{4}\left(\sqrt{3}M_1 + \sqrt{M_2^2 + 2M_1M_2}\right) - M_1$$

$$\therefore x = \frac{3M_1}{4} - M_1 + \frac{1}{4}\sqrt{3M_2^2 + 6M_1M_2}$$

$$\therefore x = \frac{1}{4}\left(\sqrt{3M_2^2 + 6M_1M_2} - M_1\right)$$

Thus, we can now characterize the segment (r+x):

$$(r + x) = \frac{M_1}{2} - \frac{M_1}{4} + \sqrt{3M_2^2 + 6M_1M_2} = \frac{1}{4}\left(M_1 + \sqrt{3M_2^2 + 6M_1M_2}\right)$$

We have now characterized the necessary components to solve for s:

$$s^2 = (r + x)^2 + \left(\frac{Q}{2}\right)^2 = (r + x)^2 + \frac{Q^2}{4}$$

Processing the components, we get:

$$(r + x)^2 = \frac{1}{16}\left(M_1^2 + 2M_1\sqrt{3M_2^2 + 6M_1M_2} + 3M_2^2 + 6M_1M_2\right)$$

$$\frac{Q^2}{4} = \frac{1}{16}\left(\sqrt{3}M_1 + \sqrt{M_2^2 + 2M_1M_2}\right)^2$$

$$\therefore \frac{Q^2}{4} = \frac{1}{16}\left(3M_1^2 + 2M_1\sqrt{3M_2^2 + 6M_1M_2} + M_2^2 + 2M_1M_2\right)$$

Adding, we get:

$$s^2 = \frac{1}{16}\left(4M_1^2 + 4M_1\sqrt{3M_2^2 + 6M_1M_2} + 4M_2^2 + 8M_1M_2\right)$$

$$\therefore s^2 = \frac{1}{4}\left(M_1^2 + 2M_1M_2 + M_2^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}\right)$$

And $s = \frac{1}{2}\sqrt{(M_1 + M_2)^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}}$

Finally:

$$AF_D \geq M_2 + \sqrt{(M_1 + M_2)^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}}$$

4.5 Interference Fit

Interference occurs with the following equality:

$$\left(1 + \sqrt{3}\right)M_1 = M_2 + \sqrt{(M_1 + M_2)^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}}$$

Re-arranging, we get:

$$\left[\left(1 + \sqrt{3}\right)M_1 - M_2\right]^2 = (M_1 + M_2)^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}$$

$$\left(4 + 2\sqrt{3}\right)M_1^2 - \left(2 + 2\sqrt{3}\right)M_1M_2 + M_2^2 = M_1^2 + 2M_1M_2 + M_2^2 + M_1\sqrt{3M_2^2 + 6M_1M_2}$$

$$\sqrt{3M_2^2 + 6M_1M_2} = \left(3 + 2\sqrt{3}\right)M_1 - \left(4 + 2\sqrt{3}\right)M_2$$

$$3M_2^2 + 6M_1M_2 = \left[\left(3 + 2\sqrt{3}\right)M_1 - \left(4 + 2\sqrt{3}\right)M_2\right]^2$$

$$3M_2^2 + 6M_1M_2 = \left(21 + 12\sqrt{3}\right)M_1^2 - \left(48 + 28\sqrt{3}\right)M_1M_2 + \left(28 + 16\sqrt{3}\right)M_2^2$$

Grouping terms, we arrive at a quadratic which must be solved for M_2:

$$\left(25 + 16\sqrt{3}\right)M_2^2 - \left(54 + 28\sqrt{3}\right)M_1M_2 + \left(21 + 12\sqrt{3}\right)M_1^2 = 0$$

$$M_2 = \frac{\left(54+28\sqrt{3}\right)M_1 \pm \sqrt{\left(54+28\sqrt{3}\right)^2 M_1^2 - 4\left(25+16\sqrt{3}\right)\left(21+12\sqrt{3}\right)M_1^2}}{2\left(25+16\sqrt{3}\right)}$$

$$M_2 = \frac{\left(27+14\sqrt{3}\right)M_1 \pm 2\sqrt{\left(54+30\sqrt{3}\right)}M_1}{\left(25+16\sqrt{3}\right)}$$

$$\therefore M_2 = \left(0.972 \pm 0.391\right)M_1$$

Since M_2 is less than M_1 at the Interference Fit point, we have:

$$M_2 \leq 0.581M_1$$

Appendix 1: Regular Polygons

As discussed in Part I, Section 5.1, a critical parameter in the analysis is the relationship that exists between the number of sides in a regular polygon and the sum of its interior angles. The following provides a proof for the fact that the sum of the interior angles for an n-sided polygon is equal to $180°(n-2)$.

Figure A1 illustrates an n-sided polygon with center point A and vertices V_1 to V_n.

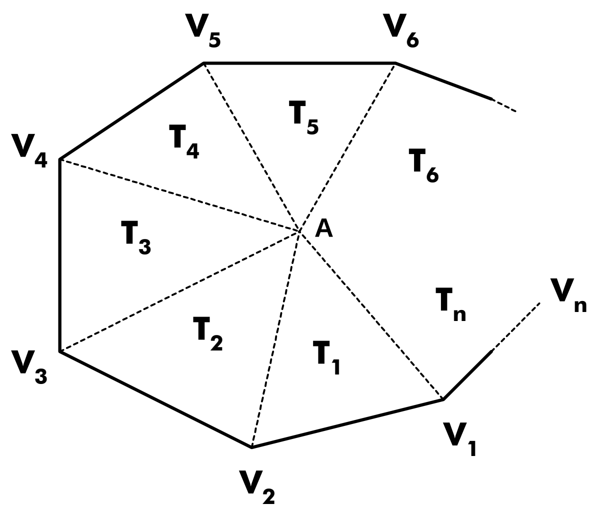

Figure A1: An N-Sided Regular Polygon

We can readily see that for every side there is an internal triangle associated with it, comprised of an external polygon side and the dashed lines connecting the end points of the side (vertices) to the center point A. It is evident that an n-sided polygon has n internal triangles.

We know that the sum of the internal angles in each of the triangles is 180°. Therefore the sum of all the angles in the n triangles is $n \cdot 180°$.

To find the sum of just the interior angles of the polygon (the angles formed at the vertices), we need to add up all the angles at the vertices. We can get this value from the sum of the angles in the n triangles, but using this sum requires us to subtract out the angles formed at the apex of each triangle, located at the center point A. This can be easily done because we know that the sum of all the triangle apex angles is 360°. We know this because all of these apex angles are encompassed in one revolution around the polygon.

Let's then define the sum of the interior angles of the Polygon as $I_{\angle}$.

Therefore $I_{\angle} = n \cdot 180° - 360° = 180°(n-2)$

A distinguishing feature of Regular Polygons is the fact that all of the interior angles are equal in magnitude. Therefore we can easily find an expression that defines this value by dividing the sum of the interior angles by the number of angles as follows:

$$|\angle_V| = \frac{I_\angle}{n} = \frac{180°(n-2)}{n}$$

As we observed in Part 1, Section 5.1, we're interested in half the value of the vertex angle, which we defined as θ.

So $\theta = \frac{180°(n-2)}{2n} = \frac{90°(n-2)}{n}$

Table A1 below summarizes the key characteristics for the first few Regular Polygons:

Table A1: Regular Polygon Characteristics

Polygon	# of Sides	Sum of Interior Angles	Magnitude of Interior Angle
Triangle	3	180°	60
Square	4	360	90
Pentagon	5	540	108
Hexagon	6	720	120
Heptagon	7	900	128.57
Octagon	8	1080	135